Reprint

ISSN 0973-9424

International Journal Of Mathematical Sciences And Engineering Applications

(IJMSEA)

ASCENT PUBLICATION @ ASCENT PUNE, INDIA

www.ascent-journals.com

International J. of Math. Sci. & Engg. Appls. (IJMSEA)
ISSN 0973-9424, Vol. 17 No. II (December, 2023), pp. 1-8

FUZZY GENERALISED LATTICE ORDERED GROUPS (Fuzzy gl-groups)

PARIMI RADHAKRISHNA KISHORE[1] AND GEBRIE YESHIWAS TILAHUN[2]

[1] Parimi Radhakrishna Kishore,
Associate Professor, Department of Mathematics,
Arba Minch University, Arba Minch, Ethiopia
and also Guest Faculty, Department of Mathematics,
SRM University-AP, Andhra Pradesh, PIN Code: 522240, India
Email: parimirkk@gmail.com
[2] Gebrie Yeshiwas Tilahun,
Ph.D-Research Scholar, Department of Mathematics,
Arba Minch University, Arba Minch, Ethiopia
Email: gebrieyeshiwas0@gmail.com

Abstract

A generalised lattice ordered group (gl-group) is a system in which the underlying set is a generalised lattice as well as a group. This paper, deals with the concept of L-fuzzy gl-subgroup of a gl-group. Introduced the concept of L-fuzzy gl-subgroup and charecterized that by its level subsets. Later, discussed about images and pre-images of L-fuzzy gl-subgroups under a gl-homomorphism. Finally discussed about equivalency of the direct product of L-fuzzy gl-subgroups with its components.

Key Words and Phrases : *Group, Poset, Lattice, Fuzzy set, Fuzzy group, Fuzzy lattice.*

2000 AMS Subject Classification : 06-XX .

Corresponding author (parimirkk@gmail.com)

1. Introduction

The theory of lattice ordered groups (l-groups) is well known from the books [15, 16] and the concept of fuzzy lattice ordered group introduced and developed by Saibaba in [17]. Murty and Swamy [6] introduced the concept of generalised lattice and the theory of generalised lattices developed by the author Kishore in [7, 8] that can play an intermediate role between the theories of lattices and posets. The concepts and the corresponding theory of fuzzy generalised lattices [9, 10, 11], generalised lattice ordered groups (gl-groups) [12, 13, 14] introduced and developed by the author Kishore. This paper, deals with the concept of L-fuzzy gl-subgroup of a gl-group. Section 2 contains some preliminaries from the references. In Section 3, introduced the concept L-fuzzy gl-subgroup of a gl-group and charecterized that by its level subsets. Also proved that the set of all L-fuzzy gl-subgroups of a gl-group is a complete lattice. In section 4, discussed about the images and pre-images of L-fuzzy gl-subgroups under a gl-homomorphism. Finally in section 5, proved that the direct product of L-fuzzy gl-subgroups of respective gl-groups is again a L-fuzzy gl-subgroup of direct product of the gl-groups.

2. Preliminaries

This section contains some preliminaries from the references those are useful in the next sections. The definitions of generalised lattice, subgeneralised lattice, homomorphism and product of generalised lattices are known from [6, 7, 8].

Definition 2.1 [Kishore [12]] : A system $(G,\ +,\ \ \leq)$ is called a generalised lattice ordered group (gl-group) if (i) $(G,\ \ \leq)$ is a generalised lattice, (ii) $(G,\ +)$ is a group and (iii) every group translation $x \to a + x + b$ on G is isotone. That is $x \leq y \Rightarrow a + x + b \leq a + y + b$ for all $a, b \in G$.

Note : Through out this paper G denotes a gl-group and 0 denotes the additive identity element of G.

Definition 2.2 [Kishore [13]] : For any $x \in G$, define $|x| = mu\{x, -x\},\ x^+ = mu\{x, 0\}$ and $x^- = mu\{-x, 0\}$.

Definition 2.3 [Kishore [14]] : Let X be a finite subset of G. Define the positive part of X by $X^+ = mu(ML(X) \cup \{0\})$.

Definition 2.4 [1] : A lattice L is said to be regular if it satisfies the following condition: for any $a, b \in L$; $a \neq 0, b \neq 0$ implies $a \wedge b \neq 0$.

Definition 2.5 [Kishore [14]] : A subgroup S of G is said to be a gl-subgroup of G if S is a subgeneralised lattice of G.

Definition 2.6 [Kishore [14]] : Let G, H be gl-groups. A group homomorphism $f : G \to H$ is said to be a gl-homomorphism if f is a homomorphism of generalised lattices.

Definition 2.7 [1] : A lattice L is said to be a complete lattice if for any subset of L the infimum and supremum exists in L.

Definition 2.8 [5] : Let X be a non-empty set and L is a complete lattice satisfying the infinite meet distributive law. Then any mapping from X into L is called a $L-$fuzzy subset of X.

Definition 2.9 [5] : Let X be a non-empty set and L is a complete lattice satisfying the infinite meet distributive law. Let μ be a L-fuzzy subset of X. Then for any $\alpha \in L$, the set $\mu_\alpha = \{x \in X \mid \mu(x) \geq \alpha\}$ is called a level subset of μ.

3. L-fuzzy gl-subgroups

In this section introduced the concept L-fuzzy gl-subgroup of a gl-group, discussed its properties and charecterized by its level subsets. Also proved that the set of all L-fuzzy gl-subgroups of a gl-group is a complete lattice.

Definition 3.1 : A fuzzy set μ in G is said to be a L-fuzzy gl-subgroup of G, if for any $x, y \in G$ and for any finite subset A of G we have (i) $\mu(x + y) \geq \mu(x) \wedge \mu(y)$ (ii) $\mu(-x) = \mu(x)$ (iii) $\mu(s) \geq \bigwedge_{a\in A} \mu(a)$ for all $s \in mu(A)$ (iv) $\mu(t) \geq \bigwedge_{a\in A} \mu(a)$ for all $t \in ML(A)$.

Theorem 3.2 : Let μ be a L-fuzzy gl-subgroup of G. Then $\mu(0) \geq \mu(x)$ for all $x \in G$. Proof: Let $x \in G$. Consider $\mu(0) = \mu(x-x) = \mu(x+(-x)) \geq \mu(x)\wedge\mu(-x)$ (by definition 3.1) $= \mu(x) \wedge \mu(x) = \mu(x)$. Therefore $\mu(0) \geq \mu(x)$ for all $x \in G$.□

Theorem 3.3 : A fuzzy set μ in G is L-fuzzy gl-subgroup of G if and only if (i) $\mu(x - y) \geq \mu(x) \wedge \mu(y)$ for all $x, y \in G$ and (ii) $\mu(s) \wedge \mu(t) \geq \bigwedge_{a\in A} \mu(a)$ for all $s \in mu(A), t \in ML(A)$ and for any finite subset A of G.

Proof : Suppose μ is L-fuzzy gl-subgroup of G. (i) Let $x, y \in G$. Consider $\mu(x - y) = \mu(x + (-y)) \geq \mu(x) \wedge \mu(-y)$ (by definition 3.1) $= \mu(x) \wedge \mu(y)$. Therefore $\mu(x - y) \geq \mu(x)\wedge\mu(y)$ for all $x, y \in G$. (ii) Let A be a finite subset of G. Let $s \in mu(A), t \in ML(A)$. By the definition 3.1, we have $\mu(s) \geq \bigwedge_{a\in A} \mu(a)$ and $\mu(t) \geq \bigwedge_{a\in A} \mu(a)$. Therefore $\mu(s) \wedge \mu(t) \geq \bigwedge_{a\in A} \mu(a)$. Conversely suppose the conditions (i) and (ii). To show that μ is L-fuzzy gl-subgroup of G : Let $x \in G$. Consider $\mu(0) = \mu(x-x) \geq \mu(x) \wedge \mu(x) = \mu(x)$ by the condition (i). Consider $\mu(-x) = \mu(0 - x) \geq \mu(0) \wedge \mu(x) = \mu(x)$ by the condition (i). Therefore $\mu(0) \geq \mu(x)$ and $\mu(-x) = \mu(x)$ for all $x \in G$. Let $x, y \in G$. Consider $\mu(x + y) = \mu(x - (-y)) \geq \mu(x) \wedge \mu(-y) = \mu(x) \wedge \mu(y)$. Let A be a finite subset of G.

Let $s \in mu(A), t \in ML(A)$. Then $\mu(s), \mu(t) \geq \mu(s) \wedge \mu(t) \geq \bigwedge_{a\in A} \mu(a)$. Therefore μ is L-fuzzy gl-subgroup of G. $\square$

Theorem 3.4 : A fuzzy set μ in G is L-fuzzy gl-subgroup of G if and only if μ_α is a gl-subgroup of G for all $\alpha \in \mu(G) \cup \{l \in L \mid \mu(0) \geq l\}$.

Proof : Suppose μ is L-fuzzy gl-subgroup of G. Let $\alpha \in \mu(G) \cup \{l \in L \mid \mu(0) \geq l\}$. To show that μ_α is a gl-subgroup of G : Let X be a finite subset of μ_α. Then $\mu(x) \geq \alpha$ for all $x \in X$ and $\bigwedge_{x\in X} \mu(x) \geq \alpha$. Let $s \in mu(X), t \in ML(X)$. Then by theorem 3.3, we have $\mu(s), \mu(t) \geq \mu(s) \wedge \mu(t) \geq \bigwedge_{x\in X} \mu(x) \geq \alpha$. That is $mu(X), ML(X) \subseteq \mu_\alpha$. Therefore μ_α is a subgeneralised lattice of G. Clearly μ_α is a subgroup of G. Hence μ_α is a gl-subgroup of G. Conversely suppose μ_α is a gl-subgroup of G for all $\alpha \in \mu(G) \cup \{l \in L \mid \mu(0) \geq l\}$. To show that μ is L-fuzzy gl-subgroup of G : Let $x, y \in G$ and $\beta = \mu(x) \wedge \mu(y)$. Clearly $\beta \in \mu(G) \cup \{l \in L \mid \mu(0) \geq l\}$ and $\mu(x), \mu(y) \geq \beta$. Then $x, y \in \mu_\beta$. By the supposition, we have $x - y \in \mu_\beta$. That is $\mu(x - y) \geq \beta = \mu(x) \wedge \mu(y)$. Let A be a finite subset of G and $\gamma = \bigwedge_{a\in A} \mu(a)$. Clearly $\gamma \in \mu(G) \cup \{l \in L \mid \mu(0) \geq l\}$ and $\mu(a) \geq \gamma$ for all $a \in A$. Then $a \in \mu_\gamma$ for all $a \in A$. That is A is a finite subset of μ_γ and by the supposition, we have $mu(A), ML(A) \subseteq \mu_\gamma$. Then $\mu(s) \wedge \mu(t) \geq \gamma = \bigwedge_{a\in A} \mu(a)$ for all $s \in mu(A)$ and $t \in ML(A)$. Therefore μ is L-fuzzy gl-subgroup of G. $\square$

Theorem 3.5 : Let μ be a L-fuzzy gl-subgroup of G. Then for any $x \in G$, we have (i) $\mu(s) \geq \mu(x)$ for all $s \in x^+$ (ii) $\mu(t) \geq \mu(x)$ for all $t \in x^-$ (iii) $\mu(r) \geq \mu(x)$ for all $r \in |x|$.

Proof : Let $x \in G$. (i) Let $s \in x^+ = mu(\{x, 0\})$ (by definition 2.2). Then $\mu(s) \geq \mu(x) \wedge \mu(0) = \mu(x)$. Therefore $\mu(s) \geq \mu(x)$ for all $s \in x^+$. (ii) Let $t \in x^- = mu(\{-x, 0\})$ (by definition 2.2). Then $\mu(t) \geq \mu(-x) \wedge \mu(0) = \mu(-x) = \mu(x)$. Therefore $\mu(t) \geq \mu(x)$ for all $t \in x^-$. (iii) Let $r \in |x| = mu(\{x, -x\})$ (by definition 2.2). Then $\mu(r) \geq \mu(x) \wedge \mu(-x) = \mu(x) \wedge \mu(x) = \mu(x)$. Therefore $\mu(r) \geq \mu(x)$ for all $r \in |x|$. $\square$

Definition 3.6 : Let X be a finite subset of G. Define the negative part of X by $X^- = mu(ML(mu(-X)) \cup \{0\})$ and the modulus of X by $|X| = mu(ML(X) \cup ML(mu(-X)))$.

Theorem 3.7 : Let μ be a L-fuzzy gl-subgroup of G. Then for any finite subset X in G, we have (i) $\mu(s) \geq \bigwedge_{x\in X} \mu(x)$ for all $s \in X^+$ (ii) $\mu(t) \geq \bigwedge_{x\in X} \mu(x)$ for all $t \in X^-$ (iii) $\mu(p) \geq \bigwedge_{x\in X} \mu(x)$ for all $p \in |X|$.

Proof : Let X be a finite subset of G. (i) Let $s \in X^+ = mu(ML(X) \cup \{0\})$ (by definition 2.3). Consider $\mu(s) \geq \bigwedge_{a\in ML(X)\cup\{0\}} \mu(a) = (\bigwedge_{a\in ML(X)} \mu(a)) \wedge \mu(0) = \bigwedge_{a\in ML(X)} \mu(a)$ by theorem 3.2. Clearly $\mu(a) \geq \bigwedge_{x\in X} \mu(x)$ for all $a \in ML(X)$. Therefore $\mu(s) \geq$

$\bigwedge_{a\in ML(X)}\mu(a) \geq \bigwedge_{x\in X}\mu(x)$. (ii) Let $t \in X^{-} = mu(ML(mu(-X)) \cup \{0\})$. Consider $\mu(t) \geq \bigwedge_{b\in ML(mu(-X))\cup\{0\}}\mu(b) = (\bigwedge_{b\in ML(mu(-X))}\mu(b)) \wedge \mu(0) = \bigwedge_{b\in ML(mu(-X))}\mu(b)$ by theorem 3.2. Clearly $\mu(b) \geq \bigwedge_{c\in mu(-X)}\mu(c)$ for all $b \in ML(mu(-X))$ and also $\mu(c) \geq \bigwedge_{x\in -X}\mu(x) = \bigwedge_{x\in X}\mu(-x) = \bigwedge_{x\in X}\mu(x)$. Therefore $\mu(t) \geq \bigwedge_{b\in ML(mu(-X))}\mu(b) \geq \bigwedge_{c\in mu(-X)}\mu(c) \geq \bigwedge_{x\in X}\mu(x)$. (iii) Since $\mu(a) \geq \bigwedge_{x\in X}\mu(x)$ for all $a \in ML(X)$, we have $\bigwedge_{a\in ML(X)} \geq \bigwedge_{x\in X}\mu(x)$. Since $\mu(a) \geq \bigwedge_{c\in mu(-X)}\mu(c) \geq \bigwedge_{x\in X}\mu(x)$, we have $\bigwedge_{a\in ML(mu(-X))}\mu(a) \geq \bigwedge_{x\in X}\mu(x)$. Therefore $\mu(p) \geq \bigwedge_{a\in ML(X)\cup ML(mu(-X))}\mu(a) = (\bigwedge_{a\in ML(X)}\mu(a)) \cap (\bigwedge_{a\in ML(mu(-X))}\mu(a)) \geq \bigwedge_{x\in X}\mu(x)$ for all $p \in |X|$. □

Theorem 3.8 : Let μ be a L-fuzzy gl-subgroup of G. If the support of μ, $Supp(\mu) = \{x \in G \mid \mu(x) > 0\}$ is non-empty and L is regular, then $Supp(\mu)$ is a gl-subgroup of G. (By using the definitions 2.4 and 2.5 we can prove this).

Theorem 3.9 : The intersection of any family of $L-$fuzzy gl-subgroups of G is again a L-fuzzy gl-subgroup of G. (By using the definition 2.5 we can prove this).

Definition 3.10 : Let A be a non-empty subset of G and $A \neq G$. Define a map $\chi_A : G \to L$ by $\chi_A(x) = \{ \begin{matrix} p & if x \in A \\ q & if x \in G - A \end{matrix}$ where $p, q \in L$ and $p < q \neq 0$.

Theorem 3.11 : A is a gl-subgroup of G if and only if χ_A is L-fuzzy gl-subgroup of G.

Theorem 3.12 : Let μ be a L-fuzzy gl-subgroup of G. Then $G_\mu = \{x \in G \mid \mu(x) = \mu(0)\}$ is a gl-subgroup of G.

Proof : To show that G_μ is a subgroup of G : Since $0 \in G_\mu$, G_μ is non-empty. Let $x, y \in G_\mu$. Then $\mu(x) = \mu(0) = \mu(y)$. This implies $\mu(x+y) \geq \mu(x) \wedge \mu(x) = \mu(0)$. Since by theorem 3.2 we have $\mu(0) \geq \mu(x+y)$, we get $\mu(x+y) = \mu(0)$ and then $x + y \in G_\mu$. Also since $\mu(-x) \geq \mu(x) = \mu(0)$, we have $\mu(-x) = \mu(0)$ and then $-x \in G_\mu$. Therefore G_μ is a subgroup of G. To show that G_μ is a subgeneralised lattice of G : Let X be a finite subset of G_μ. For any $s \in mu(X)$, we have $\mu(s) \geq \bigwedge_{x\in X}\mu(x) = \mu(0)$. Similarly for any $t \in ML(X)$, we have $\mu(t) \geq \bigwedge_{x\in X}\mu(x) = \mu(0)$. Which implies $\mu(s) = \mu(0) = \mu(t)$ for all $s \in mu(X)$ and $ML(X)$. That is $mu(X), mu(X) \subseteq G_\mu$. Therefore G_μ is a subgeneralised lattice of G. Hence the result follows from definition 2.5. □

Definition 3.13 : Let μ be a L-fuzzy subset of G. Then the smallest L-fuzzy gl-subgroup of G containing μ is called L-fuzzy gl-subgroup of G generated by μ, denoted by $< \mu >$.

Theorem 3.14 : The set of all L-fuzzy gl-subgroups of G is a complete lattice under set inclusion.

4. L-fuzzy gl-subgroups under gl-homomorphism

In this section discussed about the images and pre-images of L-fuzzy gl-subgroups under a gl-homomorphism.

Theorem 4.1 : Let G, G' be gl-groups. Let μ, ν be L-fuzzy gl-subgroups of G, G' respectively. Let $f : G \to G'$ be a gl-homomorphism and onto. Then we have (i) $f(\mu)$ is L-fuzzy gl-subgroup of G' provided that μ has sup-property (ii) $f^{-1}(\nu)$ is L-fuzzy gl-subgroup of G (iii) $f(\mu)(0') = \mu(0)$ where $0' \in G'$ and $0 \in G$ (iv) $f(G_\mu) \subseteq G'_{f(\mu)}$ (v) If μ is constant on $Kerf$, then $f(\mu)(f(x)) = \mu(x)$ for all $x \in G$ (vi) $f^{-1}(G'_\nu) \subseteq G_{f^{-1}(\nu)}$.

Proof : (i) Suppose μ has sup-property. To show that $f(\mu)$ is $L-$fuzzy gl-subgroup of G' : Clearly $f(\mu)$ is $L-$fuzzy subgroup of G'. Let $Y = \{y_1, y_2, \cdots, y_n\}$ be a finite subset of G'. Since f is onto, for each $y_i \in Y$ there exists $x_i \in G$ such that $f(x_i) = y_i$, that is $x_i \in f^{-1}(y_i)$. Since μ has sup-property and $f^{-1}(y_i) \subseteq G$; there exists $z_i \in f^{-1}(y_i)$ (that is $f(z_i) = y_i$) such that $\mu(z_i) = f(\mu)(y_i)$. By definition 2.6, we have $ML(Y) = ML(\{y_i \mid 1 \le i \le n\}) = ML(\{f(z_i) \mid 1 \le i \le n\}) = f(ML(\{z_i \mid 1 \le i \le n\}))$. Let $t \in ML(X)$. Then there exists $r \in ML(\{z_i \mid 1 \le i \le n\})$ such that $t = f(r)$. Now consider $f(\mu)(t) = Sup\{\mu(z) \mid z \in f^{-1}(t)\} = Sup\{\mu(z) \mid f(z) = t = f(r)\} \ge \mu(r) \ge \bigwedge_{i=1 \ to \ n} \mu(z_i)$ (by definition 2.1) $= \bigwedge_{i=1 \ to \ n} f(\mu)(y_i)$. Therefore $f(\mu)(t) \ge \bigwedge_{i=1 \ to \ n} f(\mu)(y_i)$ for all $t \in ML(Y)$. Similarly we can prove that $f(\mu)(s) \ge \bigwedge_{i=1 \ to \ n} f(\mu)(y_i)$ for all $s \in mu(Y)$. (ii) To show that $f^{-1}(\nu)$ is $L-$fuzzy gl-subgroup of G : Clearly $f^{-1}(\nu)$ is $L-$fuzzy subgroup of G. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a finite subset of $f^{-1}(G')$. Then there exists $y_i \in G'$ such that $x_i \in f^{-1}(y_i)$ for $1 \le i \le n$. This implies $X \subseteq f^{-1}\{y_i \mid 1 \le i \le n\}$. Let $t \in ML(X)$. Then by definition 2.6 we have $f(t) \in ML(f(X)) = f(ML(X))$. Consider $f^{-1}(\nu)(t) = \nu(f(t)) \ge \bigwedge_{i=1 \ to \ n} \nu(f(x_i)) = \bigwedge_{i=1 \ to \ n} f^{-1}(\nu)(x_i)$. That is $f^{-1}(\nu)(t) \ge \bigwedge_{i=1 \ to \ n} f^{-1}(\nu)(x_i)$ for all $t \in ML(X)$. Similarly $f^{-1}(\nu)(s) \ge \bigwedge_{i=1 \ to \ n} f^{-1}(\nu)(x_i)$ for all $s \in mu(X)$. Therefore $f^{-1}(\nu)$ is L-fuzzy gl-subgroup of G. (iii) We know that 0 is additive identity element of G. Suppose $0'$ is additive identity element of G'. Since f is onto map, there exists $x \in G$ such that $f(x) = 0'$. Since f is a group homomorphism, we have $f(0) = 0'$. Which implies $0, x \in f^{-1}(0')$. Consider $f(\mu)(0') = Sup\{\mu(x) \mid x \in f^{-1}(0')\} = Sup\{\mu(x) \mid f(x) = 0' = f(0)\} = \mu(0)$ (by theorem 3.2). (iv) By (iii) and theorem 3.12 we can prove this. (v) Suppose μ is constant on $Kerf$. Then since $f(0) = 0'$ (that is $0 \in Kerf$), we have $\mu(x) = \mu(0)$ for all $x \in Kerf$. To show that for all $x \in G$, $f(\mu)(f(x)) = \mu(x)$: Let $x \in G$.

Consider $f(\mu)(f(x)) = Sup\{\mu(z) \mid z \in f^{-1}(f(x))\} = Sup\{\mu(z) \mid f(z) = f(x)\}$. Observe that $f(z) = f(x) \Rightarrow z - x \in Kerf \Rightarrow \mu(z-x) = \mu(0)$ and similarly $f(z) = f(x) \Rightarrow x - z \in Kerf \Rightarrow \mu(x-z) = \mu(0)$. Now consider $\mu(z) = \mu(z-x+x) \geq \mu(z-x) \wedge \mu(x) = \mu(0) \wedge \mu(x) = \mu(x)$ and similarly we can prove $\mu(x) \geq \mu(z)$. Therefore we proved that $f(z) = f(x) \Rightarrow \mu(x) = \mu(z)$. Which implies $f(\mu)(f(x)) = \mu(x)$. (vi) By v) and theorem 3.12 we can prove this. □

Corollary 4.2 : If μ is constant on $Kerf$ then we have (i) $f^{-1}(f(\mu)) = \mu$ (ii) $f(f^{-1}(\nu)) = \nu$.

5. Product of L-fuzzy gl-subgroups of gl-groups

In this section defined direct product of $L-$fuzzy subsets of gl-groups and discussed about equivalency of the direct product of L-fuzzy gl-subgroups with its components.

Definition 5.1 : Let G_i be gl-groups and μ_i be L-fuzzy subset of G_i for $1 \leq i \leq n$. Define the map $\mu = \mu_1 \times \cdots \times \mu_n : G = G_1 \times \cdots \times G_n \to L$ by $\mu(x_1, \cdots, x_n) = \mu(x_1) \wedge \cdots \wedge \mu(x_n)$. Then μ is an L-fuzzy subset of G, called direct product of $\mu_1, \cdots, \mu_n$.

Proposition 5.2 : If each μ_i is L-fuzzy gl-subgroup of G_i for $1 \leq i \leq n$ then $\mu = \mu_1 \times \cdots \times \mu_n$ is an L-fuzzy gl-subgroup of $G = G_1 \times \cdots \times G_n$.

Proposition 5.3 Let G_i be gl-groups and μ_i be L-fuzzy subset of G_i with greatest element $\mu_i(0)$ of L for $1 \leq i \leq n$. If $\mu = \mu_1 \times \cdots \times \mu_n$ is an L-fuzzy gl-subgroup of $G = G_1 \times \cdots \times G_n$, then each μ_i is a L-fuzzy gl-subgroup of G_i.

Acknowledgement

This paper dedicated to the professor: (late) Prof. Mellacheruvu Krishna Murty, Professor, Department of Mathematics, Andhra University, Visakhaptnam - 530003, INDIA and also to the purohith: (late) Sri. Parimi Lakshmi Narasimha Prasad, Purohith, Karlapalem village, Karlapalem mandal, Guntur district, PIN Code: 522111, Andhra Pradesh state, INDIA.

REFERENCES

[1] Birkhoff G., Lattice theory. Amer. Math. Soc. Colloq. Publ. XXV, Providence, R.I, (1967).

[2] Rosenfeld A., Fuzzy groups, J. Math. Anal. Appl., 35 (1971) 512-517.

[3] Yuan Bo., Wu W., Fuzzy ideals on a distributive lattice, Fuzzy Sets and Systems, 35 (1990) 231-240.

[4] Ajmal N. and Thomas K. V., Fuzzy lattices. Information sciences, 79 (1994) 271-291.

[5] John N Mordeson and Malik D. S., Fuzzy Commutative Algebra. World Scientific publishing. co.pvt.Ltd., (1998).

[6] Murty M. K. and Swamy U. M., Distributive partially ordered sets. The Aligarh Bull.of Math, 8 (1978) 1-16.

[7] Kishore P. R., Murty M. K., Ramani V. S. and Patnaik M. D. P., On generalised lattices. Southeast Asian Bulletin of Mathematics, 33 (2009) 1091-1104.

[8] Kishore P. R., The lattice of convex subgeneralised lattices of a generalised lattice. International Journal of Algebra, 3(17) (2009) 815-821.

[9] Kishore P. R., Fuzzy generalised lattices. International Journal of Mathematical Sciences & Enggineering Applications (IJMSEA), 9(3) (2015) 1-9.

[10] Kishore P. R., Homomorphism on Fuzzy generalised lattices. International Journal of Contemporary Mathematical Sciences, 11(6) (2016) 275-279.

[11] Kishore P. R. and Melesse M. H., Fuzzy Ideals and Fuzzy Congruences of a Generalised Lattice. International Journal of Mathematics And its Applications, 8(3) (2020), 187-191.

[12] Kishore P. R., Generalised lattice ordered groups (gl-groups). International Journal of Algebra, 7(2) (2013), 63-68.

[13] Kishore P. R. and Kifetew D. C., Properties of generalised lattice ordered groups. International journal of computing science and applied mathematics, 7(1) (2021), 25-27.

[14] Kishore P. R. and Korma S. G., gl-subgroups and isomorphism theorems of gl-groups, International Journal of Mathematics And its Applications (IJMAA), 11(3)(2023), 93-101.

[15] Kopytov V. M., Medvedev N. Ya., The theory of lattice ordered groups. Springer-Science+Business Media, B.V., 1994.

[16] Stuart A. Steinberg, Lattice ordered rings and modules. Springer-Science+Business Media, LLC 2010.

[17] Satya Saibaba G. S. V., Fuzzy lattice ordered groups, Southeast Asian Bulletin of Mathematics, 32 (2008) 749-766.

[18] Gabor Szasz (Professor of Mathematics at Szeged University), Introduction to Lattice Theory, Academic Press New Yark and London, 1963.